The option value of government guarantees in infrastructure projects

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The participation of private capital in public infrastructure investment projects has been sought by many governments who perceive this as a way to overcome budgetary constraints and foster economic growth. For some types of projects, this investment may require government participation in the form of project guarantees in order to reduce the risk to the private investor, and as a consequence, the government assumes a contingent liability which may have significant future budgetary impacts. We present a minimum traffic guarantee (MTG) real options model that differs from most of the literature in the field by using market data to determine stochastic project parameters. This model can be used to assess the value of these guarantees, allows the government to analyse the cost–benefit of each level of support, and proposes an alternative to limit the exposure of the government while still maintaining the benefits to the private investor. We apply this model to the projected 1000 mile long BR-163 toll road that will link the Brazilian Midwest to the Amazon River. We conclude that the use of public–private partnerships (PPP) with guarantees and caps on total government outlays can be modelled effectively using option pricing methods and can be a solution to attract private investment to high risk public infrastructure projects.

Keywords: Real options, infrastructure projects, toll roads, government guarantees, concessions.

Introduction

The 1990s were characterized by a worldwide trend towards an increase in participation of private investment in public infrastructure projects, the main motivation being the gains in efficiency derived from the substitution of public administration for private enterprises, a better allocation of risk and budgetary constraints of governments. On the other hand, private infrastructure projects are subject to government regulation, cover services deemed essential by society, require large amounts of irreversible capital investment, have long maturity time and are usually offered monopolistically. This combination of factors ensures that once implemented, the interests of the government and those of the private investor begin to diverge, which subjects these projects to pressure from users and opportunist behaviour by the government, thereby increasing the risk to the investor. As a consequence, private investors may demand that the government provide guarantees that have the effect of reducing these risks.

Government guarantees have been used frequently in private infrastructure projects. For the expansion of the gas-fired energy plant of Barranquilla at a cost of $755 million, the Colombian government guaranteed that the state-owned Public Utility Company would honour a \textit{take or pay} contract (Beato, 1997; Lewis and Mody, 1998). The concession of the Santiago–Valparaiso–Viña del Mar toll road in 1998, with 130km and a cost of $400 million, offered a minimum traffic guarantee at an additional cost to the investor (Engel et al., 2000). The Linha Amarela Expressway in Rio de Janeiro, in 1994, also includes a grant of US$112 million, for a total project value of US$174 million (Dailami and Klein, 1997).

The presence of the government as a risk reduction agent may be necessary since it controls many of the variables that affect the project, such as interest rates, regulation requirements, political risk, etc., or because market risk is such that the project is unable to attract private investment. The Costanera Norte toll road in
Chile, for example, had no bidders when it was first auctioned out in 1998. Only in 2002, after government supports were included was the road successfully bid.

By offering guarantees for infrastructure projects, the government becomes responsible for all future liabilities that these supports may cause, which may be very onerous to the government if the risks involved are not adequately analysed and quantified. The foreign exchange guarantees provided by the Spanish government in the 1970s and the failure of the Mexican toll road concessions after the 1994 Mexican crisis eventually cost $2.5 billion and $8.9 billion respectively to these governments. Thus, the importance of the valuation of these supports is that it allows the government not only to determine the value of budgetary and fiscal impacts of future contingent liabilities, but also to define levels of guarantees that are high enough to allow the project to be economically feasible but low enough not to burden the government and society in excess.

Government guarantees allow the private investor to recoup part of its losses. If a project underperforms in a particular year, the investor has the option to demand that the government reimburse him the shortfall, up to a pre-established level of guarantee. Owing to these characteristics, the valuation of these guarantees requires the use of option pricing methods known as real options analysis (Dixit and Pindyck, 1994; Trigeorgis, 1996).

We develop a real options model for infrastructure projects subject to minimum traffic guarantee (MTG) in order to assess the value of these guarantees, their impact on risk of the project and the expected value of the government outlays. We also show how different levels of guarantees affect the project’s risk profile and provide suggestions on how the government may use this information to minimize its costs.

The literature on the use of real option valuation methods for infrastructure projects is limited. Rose (1998) uses real option analysis to show that the value of the Melbourne Central Toll project in Australia increases considerably when the flexibility to increase revenues is considered. Brandão (2002) applied real option valuation to the Via Dutra highway in Brazil which incorporates the value of the options to expand and to abandon. Ng and Björnsson (2004) present arguments in favour of the use of real option approach to the analysis of a toll road concession project. Bowe and Lee (2004) analyse the Taiwan High-Speed Rail project, where the concessionaire has the option to develop real estate projects along the right of way and show that the value of these options can constitute a very significant proportion of total project value. Pimentel et al. (2007) investigate the optimal timing of investment for a high speed rail project.

Charoenpornpattana et al. (2002) address the problem of valuing government supports and model a minimum revenue guarantee (MRG) and shadow toll as a bundle of independent options. Lewis and Mody (1998) provide a discussion of the use of option pricing methods (contingent claims analysis) to value several types of guarantees that were given in the El Cortijo–El Vino toll road concession in Colombia. More recently, Cheah and Liu (2006) propose a real options model to value a MRG using a Monte Carlo simulation approach and apply this to the case of the Malaysia–Singapore Second Crossing. Huang and Chou (2006) also use a real options approach to value a MRG in a project that has an option to abandon using the Taiwan High-Speed Rail project as a numerical example, and show that both the government supports and the option to abandon create value. Chiara et al. (2007) develop a model where the MRG is in the form of Bermudan and Australian options which is solved using a least squares simulation approach and apply this to a hypothetical case. All these authors adopt the minimum revenue guarantee (MRG) model which solves for the option value by discounting project revenues at the risk free rate.

The underlying assumption of the MRG model is that project revenues are a marketed asset whose risk premium can be directly observed in the market, allowing for the use of the risk free rate for option valuation. Given that revenues are simply the product of traffic volume and the toll rate constant, it is clear that uncertainty in revenues is caused solely by uncertainty about future traffic levels, which is not a marketed asset. Thus, the problem with this approach is that the risk premium cannot be observed in the market and there is no guarantee that the risk free rate is the appropriate one to use in the MRG model.

In contrast to previous studies, we adopt a minimum traffic guarantee (MTG) model rather than the standard MRG model, where the underlying asset is the traffic volume level, and show how the risk premium of the traffic level (and of the revenues) can be determined indirectly from the project returns, allowing for a more accurate assessment of the model parameters. We also model situations where specific limits on government exposure are set by establishing caps on the total outlay derived from the concession of supports, thus ensuring that a specific amount of subsidy will not be surpassed. This allows the government to limit its future liability for any supports that may have been granted.

In the following section we discuss possible types of government infrastructure concessions projects and their impact on private sector risk. In the third section, we present a minimum traffic guarantee (MTG) valuation model for private infrastructure projects,
and in the next section we illustrate this with an application to the BR-163 toll road project. In the final section we conclude.

**Toll road concession models**

Toll road concession contracts can be classified according to the degree of risk the private investor is subjected to.

Low risk projects may benefit from the traditional concession model where the government provides no supports and holds no future liabilities and all risks are borne by the concessionaire. This is the most widely used type of concession, prevalent in Argentina, Brazil, Chile and the United States, and is based on the build-operate-transfer (BOT) model (Bousquet and Fayard, 2001; Hammami et al., 2006). According to the Kikeri and Burman (2007), more than 160 such projects totalling $37 billion of concessions were granted in Latin America and the Caribbean between 1990 and 2005. In the United States, 4000 miles of toll road concessions for the US$100 billion Trans Texas Corridor and portions of the highway I-35 are currently under construction or being auctioned (Persad et al., 2004). This model generally breaks down when the project risk is deemed so high or the returns so uncertain that the government is unable to attract private capital for the project. This typically happens because governments usually grant out concessions of the more profitable projects first, and after this initial stock is depleted, it is left with less attractive low return projects of higher risk.

High risk projects may require some level of government support that reduces the risk and/or increases the returns to the private investor by means of government participation in private infrastructure investment, also known as public–private partnerships (PPP). Although PPPs in some countries refer to any private investment in public infrastructure such as BOTs, we will use it to refer to a class of high risk projects where there is some form of government support and risk sharing and which usually operate under a legal framework that is distinct from that of the standard concession model, as is the case of Brazil. Under a government support PPP model, for example, if the returns of the project are much lower than expected, the project may receive a government subsidy proportional to the reduction in the observed demand, so that a minimum level of return is maintained. Other options may also be present, such as the option to extend (or contract) the concession period, or to postpone payments due to the government. On the other hand, PPPs require a long-term commitment of the government to a project, along with the risk of taking on future liabilities that are usually not sufficiently accounted for or adequately quantified. The indiscriminate granting of government supports can become a heavy burden to society since these options create future liabilities and potential responsibilities. We analyse the case of a high risk project under a PPP where the government grants a minimum traffic guarantee in order to reduce project risk and attract private investment.

**Minimum traffic guarantee (MTG) model**

**Risk premium of traffic uncertainty**

Toll road projects offer many distinct sources of risk to the investor (Fishbein and Babbar, 1996). Many of these are private, diversifiable risks, which are of less concern to an adequately diversified investor, such as construction and political risk, or risks that can be hedged away, even if at a cost, such as exchange rate risk. On the other hand, the uncertainty over the future levels of demand for traffic on the completed road is of great consequence and constitutes an undiversifiable market risk.

We assume there is a contractual guarantee where the government is obligated to make certain payments to the concessionaire whenever the traffic level (AADT—average annual daily traffic) falls below a pre-established floor during a period of time, and that the toll rate is constant throughout the concession period.

Let $R_t$ be the observed revenue of the project ($R_t = AADT_t \times \text{Toll Rate}$) in year $t$ and $P_t$ the minimum revenue guaranteed by the government in that year. Since we assume that the toll rate is constant, revenues will follow the same stochastic process as the traffic uncertainty. Considering the minimum traffic guarantee, the effective revenue for the concessionaire in year $t$ will be $R(t) = \max (R_t, P_t)$. Similarly, the value $G(t)$ of the government guarantee in that year will be $G(t) = \max (0, P_t - R_t)$.

Given the uncertainty about the future level of traffic, we assume that traffic and revenues will vary stochastically in time following a geometric Brownian motion (GBM), as is standard in the literature. This model implies that the revenue can never be negative and that its volatility is constant in time and can be represented as:

$$dR = \mu R dt + \sigma_R R dz$$

where $dR$ is the incremental change in revenue during a short period of time $dt$, $\mu$ is the revenue growth rate in a
short interval of time \( dt \), \( \sigma_R \) is the volatility of the revenue and \( dz = \sqrt{dt} \), where \( z \sim N(0, 1) \) is the standard Wiener process. This GBM can be represented by the stochastic evolution of the returns, as shown in Equation 2, which can be discretely modelled in yearly periods as a function of the value in the previous period, as shown in Equation 3.

\[
\text{dln} R = \left( x - \frac{\sigma^2 R}{2} \right) dt + \sigma_R dz \\
R_{t+1} = R_t e^{\left( x - \frac{\sigma^2 R}{2} \right) \Delta t + \sigma_R \sqrt{\Delta t}}
\]  

(2)  

(3)

This process can be completely specified considering only its initial value \( R_0 \), a yearly growth rate and the volatility of the process, which we assume to be constant during the concession period, where Equation 1 represents the ‘true’ process of the evolution of the project revenues. To value the guarantees, on the other hand, we must use a risk neutral process where we subtract the risk premium from the expected return rate of the underlying asset, which for a marketed asset is the equivalent of substituting its ‘true’ return by the risk free rate of return.

Given that markets are incomplete for traffic and revenues, the appropriate risk premium cannot be determined directly. Some authors, such as Irwin (2003) and Dixit and Pindyck (1994) suggest an exogenous solution where an arbitrary value for the risk premium is adopted. Under the MTG model we show that the parameters for the risk premium of the revenues can be estimated from the stochastic process of the value of the project.

Let us assume that the revenue process is defined by Equation 1. Given that the revenues represent the only source of project uncertainty, we can define the evolution of the value of the project \( V = f(R) \) subject to the same standard Wiener process \( dz \) where \( dV = \mu dt + \sigma_P V dz \) where \( \sigma_P \) is the project volatility. By means of an Itô process, we can define:

\[
dV = \left[ \frac{\partial V}{\partial R} \sigma_R^2 + \frac{\partial V}{\partial \tau} + \frac{1}{2} \frac{\partial^2 V}{\partial \tau^2} \sigma_R^2 R^2 \right] dt + \frac{\partial V}{\partial R} \sigma_R R dz
\]  

(4)

From the capital asset pricing model (CAPM) we have \( \mu = \lambda \beta_R (E[R_m] - r) \), where \( \mu \) and \( \beta_R \) are respectively the risk adjusted discount rate and the beta of the project. The risk premium of \( V(R) \) is then given by \( \mu - r = \beta_R (E[R_m] - r) \). The risk premium of the project can also be expressed as \( \lambda \sigma_p \), therefore we have:

\[
\mu - r = \lambda \sigma_p
\]  

(5)

Substituting Equation 4 into Equation 5 we remain with:

\[
\frac{\partial V}{\partial R} \sigma_R^2 + \frac{\partial V}{\partial \tau} + \frac{1}{2} \frac{\partial^2 V}{\partial \tau^2} \sigma_R^2 R^2 \right] dt + \frac{\partial V}{\partial R} \sigma_R R dz
\]  

(6)

Equation 6 is the differential equation that the value of a project subject to revenue risk must conform to. With this equation we can then determine the value of options on revenues or project value, as long as we use a risk neutral process for the project revenues, with a drift rate of \( x - \lambda \sigma_R \) instead of \( a \). Under the assumption that the value of the project without options is the best unbiased estimate of its market value, from CAPM we can determine the risk premium of the project cash flows. If \( \mu \) is the expected rate of return of the project and \( \beta_R \) is its beta, then \( \mu = r + \beta_R (E[R_m] - r) \) and the project risk premium will be \( \mu - r = \beta_R (E[R_m] - r) \). Similarly, the risk premium of the revenues is given by

\[
\mu - r = \beta_R (E[R_m] - r)
\]  

(7)

We define the market price of risk \( \lambda R \) as

\[
\lambda R = \frac{\mu - r}{\sigma_R}
\]  

(8)

Substituting Equation 8 and the value of \( \beta_R = \frac{\sigma_{m,R}}{\sigma_m} \) into Equation 7, multiplying both sides by \( \left( \frac{\sigma_m}{\sigma_{m,R}} \right) \) and rearranging, we obtain \( \lambda_R \sigma_R = \left( \frac{\sigma_{m,R}}{\sigma_m} \right) \left( \frac{E[R_m]}{\sigma_m} \right) \sigma_R \), where \( \rho_R = \frac{\sigma_{m,R}}{\sigma_m \sigma_R} \) represents the correlation between the change in revenues and the market returns. Finally, we remain with

\[
\lambda_R = \rho_R \left[ \frac{E[R_m]}{\sigma_m} \right] - r \right] \sigma_R
\]  

(9)

In a similar way, the market price of risk \( \lambda_P \) of the project will be

\[
\lambda_P = \rho_P \left[ \frac{E[R_m]}{\sigma_m} \right] - r \right] \sigma_R
\]  

(10)

where \( \rho_P \) represents the correlation between the project returns and the market.

Given that we assume that the only source of uncertainty of the project is its revenue, the correlation \( \rho_R \) between the changes in revenues and the market returns will be identical to the correlation \( \rho_P \) between the project returns and the market, which implies that Equation 9 = Equation 10, and \( \lambda_R = \lambda_P = \lambda \). From Equations 7 and 8 we can then obtain \( \lambda \sigma_R = \beta_R (E[R_m] - r) \), which defines the risk premium of
the revenues. In a similar fashion we can also obtain
\[ \lambda \sigma_p = \beta_p (E[R_m] - r) \] (11)

Since the value of \( \beta_R \) is unknown, we multiply both sides of Equation 11 by \( \sigma_p / \sigma_R \) and remain with Equation 12, which is the expression for the risk premium of revenues as a function of the risk premium and volatility of the project and the volatility of the revenues, all of which are known constants.

\[ \lambda \sigma_R = \beta_p (E[R_m] - r) \frac{\sigma_R}{\sigma_p} \] (12)

The risk neutral process of revenues is then:
\[ dR = (x - \lambda \sigma_R) R dt + \sigma_R R dz \] (13)

where \( \lambda \sigma_R \) is the risk premium of revenues previously determined in Equation 12. We refer the reader to Hull (2003, pp. 660–7) for further analysis of this property.

The uncertainty over future levels of traffic is one of the key parameters of the model. For existing roadways, the traffic volatility can be observed from historical traffic series. For new roadways, this volatility can be estimated under the standard assumption that traffic levels are positively correlated with GDP (Banister, 2005; OECD, 2002, pp. 143–78). Project volatility can then be computed from a Monte Carlo simulation run of the stochastic cash flows of the project adopting the criteria proposed by Brandão et al. (2005). Owing to the leverage effect of the project’s fixed costs, project volatility tends to be greater than traffic volatility, which reduces the risk premium of revenues.

### Valuation of guarantees

The valuation of the government guarantees in infrastructure projects can be modelled as a series of independent European options with maturities between 1 and 25 years. Under the risk neutral process of the revenues defined in Equation 13, the value of the guarantee options can be determined by a Monte Carlo simulation of the stochastic traffic level assuming the option will be exercised whenever revenues fall below the established minimum, which is then discounted at the risk free rate. The value of the concession with the revenue guarantee is obtained by simply repeating this analysis for each of the 25 years of the concession and adding the present value of all these options to the static value of the project, as shown in Equation 14.

\[ \text{Value of Guarantee} = \sum_{i=1}^{25} \text{Value of Option}_i \] (14)

### Application

We now apply the MTG model to the BR-163 roadway project in Brazil. The Brazilian Army Corps of Engineers initially built the BR-163 in 1973 as a simple two-lane road with wooden bridges crossing the Amazon rainforest in the South–North direction up to the Amazon River. To this day half of the extension of approximately 1000 miles between the cities of Cuiabá, in Central Brazil and Santarém to the Amazon rainforest to the north, still remains a dirt road which is closed to traffic for several months during the rainy season, and the remainder of the road is in poor condition. One-third of the Brazilian soybean crop is produced in the region and travels 1500 miles down the BR-163 and other roads to be exported through the seaports of Santos and Paranaguá. With the new road, it is expected that traffic flow will be reversed upwards towards the port of Santarém in the Amazon River, cutting down the average distance to a third.

Future traffic is difficult to estimate, since changes in commodity prices and exchange rates can significantly affect the expected export-bound traffic. In 2005, the Brazilian government tried to auction the road as a traditional concession but there were no bidders, and one of the alternatives currently under consideration is to grant some form of government support. The traffic projections data used in this article are government estimates and were obtained online from the National Department of Transportation Infrastructure (DNIT) of the Ministry of Transportation.2

The initial investment required for the project is R$966.7 million,3 plus R$1291.5 million for continuous road improvements during the life of the concession. We assume the project has a debt level of 60%, an equity cost of capital of 16% per year, and that the risk free rate is 7%, which are typical values for the Brazilian market (Brandão, 2002). The discounted cash flow analysis using the free cash flow to equity (FCFE) model provides a NPV of R$139.8 million. The volatility of the future traffic demand was estimated assuming a correlation with regional GDP. Based on data from the Institute for Economic Analysis and Research (IPEA),4 the volatility of Brazil’s Midwest GDP from 1980 to 2002 was 6.9% per year in average, and 7.0% between 1990 and 2002. We assumed a traffic volatility of 7% per year and an initial level of traffic of 96 205 equivalent daily vehicles (EDV)5 for the year zero for all 13 toll plazas of the roadway. Given that this initial traffic volume is also uncertain, we also assumed a triangular probability distribution around this value with a minimum of 67 343 and a maximum of 125 066 EDV, corresponding to a variation of ±30%.
The risk analysis of the project shows that the project NPV has a relatively high standard deviation of R$193.3 million. There is also a 24.8% probability that the project NPV will be negative (Figure 1), which may help explain why the government has been unable to attract private investment to this project.

This analysis does not incorporate the value or the impacts on the project of any form of government supports that could be offered to make it more attractive to private investors. As shown before, in this concession model, the private investor holds all the project risks and the cost to the government is zero, and therefore, the private investor will require a higher risk premium and a higher toll rate.

The simulation results indicate a volatility of 47.8%. The risk premium of the project cash flows is determined from the CAPM model 
\[
\mu - r = \beta_C (E[R_m] - r) = 8\%
\]
and from Equation 12 we obtain a value for the risk premium of traffic of \( \sigma_R = 1.32 \). Given the risk neutral process of the revenues defined in Equation 13, we determine the value of the option considering the value of exercise in each year and the total aggregate value of all options during the concession period at each level of guarantee. Since the revenue floor protects the investor against low traffic volume, the government may appropriate revenues significantly in excess of the expected value by establishing a traffic ceiling in order to prevent excessive profits. In the El Cortijo–El Vino toll road concession in Colombia, for example, it was stipulated that if traffic volume exceeded projections by more than 10%, the additional revenues associated would be deposited in a reserve fund used to cover future shortfalls in traffic volume or for road maintenance and improvements (Lewis and Mody, 1998).

The joint modelling of a traffic floor and ceiling is a case of compound options, where distinct options can be exercised over the same underlying asset. Even though they are mutually exclusive, they exist simultaneously and must be modelled as such. This can be done by assuming that the actual traffic level will fall in any of three distinct and mutually exclusive regions: below the floor, between the floor and the ceiling or above the ceiling. For the sake of simplicity, we assume that the floor and ceiling are symmetrical relative to the expected level of traffic, but other assumptions may also be adopted with ease. The revenues received by the concessionaire in each period \( t \) are given by:
\[
R(t) = \min \{ \max (R_t, P_t), T_t \}
\]
where
- \( R_t \) is the observed level of revenues,
- \( P_t \) is the level of revenues of the traffic floor,
- \( T_t \) is the level of revenues of the traffic ceiling.

Table 1 illustrates how the project value changes with each level of guarantee. A guarantee that at least 60% of the expected traffic revenue will be received by the investor, for example, increases the project value from R$139.8 million to R$206.5 million, and this value increases as the guarantee level increases. A guarantee level of 80% increases the project NPV by a factor of 2.5, which shows that the minimum traffic guarantees are an effective form of risk reduction. This increase in value represents the expected PV of the total government outlays for the project guarantees, in addition to the project cash flows received by the private investor. As also shown in Table 1, the combined effect of symmetric traffic floor and ceiling is small compared to

<table>
<thead>
<tr>
<th>Level of guarantee</th>
<th>NPV w/ MTG</th>
<th>w/ Traffic ceiling w/ cap of 600.0</th>
<th>w/ cap of 400.0</th>
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</thead>
<tbody>
<tr>
<td>0%</td>
<td>139.9</td>
<td>139.9</td>
<td>139.9</td>
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<tr>
<td>10%</td>
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</tr>
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<td>20%</td>
<td>139.9</td>
<td>138.3</td>
<td>139.9</td>
</tr>
<tr>
<td>30%</td>
<td>140.1</td>
<td>137.6</td>
<td>140.1</td>
</tr>
<tr>
<td>35%</td>
<td>141.2</td>
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<tr>
<td>40%</td>
<td>143.5</td>
<td>139.5</td>
<td>143.5</td>
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<td>45%</td>
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</tr>
<tr>
<td>50%</td>
<td>159.7</td>
<td>150.2</td>
<td>159.7</td>
</tr>
<tr>
<td>55%</td>
<td>177.7</td>
<td>167.4</td>
<td>175.3</td>
</tr>
<tr>
<td>60%</td>
<td>206.5</td>
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</tr>
<tr>
<td>65%</td>
<td>249.9</td>
<td>226.5</td>
<td>241.6</td>
</tr>
<tr>
<td>70%</td>
<td>312.4</td>
<td>278.5</td>
<td>290.1</td>
</tr>
<tr>
<td>75%</td>
<td>388.8</td>
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<td>342.1</td>
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<td>80%</td>
<td>492.0</td>
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<td>85%</td>
<td>616.6</td>
<td>524.9</td>
<td>454.9</td>
</tr>
<tr>
<td>90%</td>
<td>764.9</td>
<td>640.9</td>
<td>512.5</td>
</tr>
</tbody>
</table>

Note: Values in R$ millions (US$1 = R$1.70).
the increase in project value from the traffic floor in this case because the expected growth rates of demand for traffic beyond the first few years of the concession are relatively small.

**Effect on risk**

We can also verify the effect that MTG may have on project risk by analysing the changes in the probability distribution of the project NPV.

The traffic floor eliminates the probability of occurrence of low NPVs, and as a consequence, increases the expected NPV, while the traffic ceiling affects the project by setting a cap on the probability of the project having very high NPVs. The two opposite project options significantly reduce the variance, and consequently, the project risk, by increasingly eliminating both tails of the distribution. Figure 2 shows the effect of traffic guarantees between 60% and 90% on the distribution of the project NPV considering both the traffic floor and ceiling.

For a guarantee level of 90%, the probability of the project having a negative NPV is zero, which implies that a return above the project’s hurdle rate is ensured. In this sense, if the government chooses to provide such high levels of guarantees it may also require that the private investor significantly reduce its risk premium, or even eliminate it completely and earn the risk free rate of return given that the project becomes essentially riskless in this case. It can be noted also that at high guarantee levels, the probability that the NPV will be at the extreme ends of the interval increases significantly.

**Government liability risk**

Given that the values of the guarantees are expected values, there is a 50% probability that the actual payments be greater (or smaller), and a small probability that they will be significantly higher, which creates budgetary and liability risks for the government. We use a Monte Carlo simulation to determine the probability distribution of the expected payments in order to analyse the risk that the government be required to honour larger than expected outlays.

Figure 3 illustrates the probability distribution of a guarantee of 80%. Although the value of this guarantee

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**Figure 2** NPV distribution for MTG levels of 60%, 70%, 80% and 90% respectively
is R$352.8 million, there is a high probability that this value will be zero, but also a 5% probability that the actual government outlays will be higher than R$1242 million.\(^6\)

Government exposure can be limited by the use of caps, where the outlays cease once a pre-established ceiling is reached. This upper limit only affects the total aggregate value of the options and not the value of each option individually, except for the borderline option. With caps, the value of the option in each year is still determined as shown previously, but the cumulative sum of all government outlays is limited to the cap, as shown in Equation 15.

\[
\text{Value of Guarantee} = \min \left( \sum_{i=1}^{25} \text{Option}_i, \text{Cap} \right) \quad (15)
\]

In practice, the choice of the cap will take into account the project size, the maximum exposure the government wishes to have on the project, and its impact on the effectiveness of the MTG. Considering that the total investment cost of the project is approximately R$2.2 billion, for illustration purposes we analyse two arbitrary cap limits of R$400 million and R$600 million, which correspond approximately to 20% and 30% of the total cost of the project. In Figure 4 we see that, as expected, the impact of these caps is to reduce the value of the guarantees, since they reduce the government outlays for the project. Because the cap affects only the total outlays of highest value, which are the ones that have the lowest probability of occurring, its effect on the guarantee is limited. This way, it is possible that the cost of the cap relative to the guarantees be reasonably small relative to the benefits derived from the elimination of the uncertainty over the maximum government exposure in the project.

Table 1 presents the value of the project for guarantee levels ranging from 0 to 90%, and for different cap limits.

Figures 5 and 6 illustrate the approximate probability distribution of a guarantee of 80% considering caps of R$600 million and R$400 million, respectively. The probability that the total outlays of the government be greater than the caps is zero, although the probability that the outlays be equal to the cap increases to 23.53% and 34.28% respectively. The probability that the total outlays be zero is still 20.62% as before, for both cases.

**Conclusion**

We analysed the problem of private investment in public infrastructure and concluded that for some classes of risky projects, it may be necessary for governments to share part of the project risk by granting certain supports. One such type of support is
the minimum traffic guarantee (MTG), which provides the concessionaire with a government subsidy if traffic falls below a pre-established level. On the other hand, determining the optimal level of these guarantees cannot be done through traditional project evaluation methods and requires the use of option pricing techniques. We show how this valuation can be performed using a real options model, and how different levels of support affect both the project risk and its value. Rather than defining an exogenous discount rate for project revenues, we use an innovative model where we consider that the main source of project uncertainty is the future traffic levels, and show how the market parameters required for the risk neutral valuation can be determined.

This approach can be used by governments to model and analyse the use of guarantees for projects of interest and choose the best combination of cost and risk reduction. Less risky projects may require fewer or no guarantees, while attracting private investment for more risky projects may entail more costly guarantee structures. If we assume that these projects are of interest to society, as is the case of the BR-163 road, the failure to attract private investment will require that the project be 100% government funded, which is by far the most expensive alternative.

We also analyse the impact of these supports on government outlays and conclude that indiscriminate granting of these guarantees can create significant future contingent liabilities for the government. We show that the use of caps on the total outlays associated with a particular level of MTG can help reduce this liability risk, and owing to their asymmetric impact on project value, they may be an acceptable solution to all stakeholders involved. This would allow governments to leverage their investment capability by redirecting scarce resources away from the financing of public infrastructure investment towards providing a limited level of guarantees to a wide range of projects, as long as precautions are taken in selecting an appropriate government project portfolio. We conclude that the use of public–private partnerships (PPP) with guarantees and caps on total government outlays can be modelled effectively using option pricing methods and can be a solution to attract private investment to higher risk public infrastructure projects.

Although we analyse here only the case of a traffic guarantees, the model is flexible and can be easily extended to include other forms of guarantees, such as shadow tolls, exchange rate, debt and equity guarantees, and the least present value of revenues (LPVR) model suggested by Engel et al. (2000).

Notes

1. Brazil is the world’s largest soybean producer with a crop of 53.9 million metric tons in 2006, mostly for export.
3. The average exchange rate in 2008 was US$1=R$1.70.
5. Equivalent to a standard two axle automobile.
6. The probability distribution of the guarantees was determined from a risk neutral stochastic process, and not through the true process of traffic or revenues, so the values shown do not represent the actual probabilities of occurrence as these can only be determined from the true process. In this case this is not possible, since each iteration of the simulation has a distinct discount rate, so it is not possible to determine either the present value or the aggregate value of the options with this method. For this reason, we resorted to risk neutral valuation which provides only the risk neutral probabilities. While these are different from the true probabilities, they provide the necessary intuition for the reader to understand that the expected value of the guarantees is only an average and that there is a small probability that significantly higher probabilities may occur.

References

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